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LETTERS TO THE EDITORS

COMMENTS ON "DISPERSION MEASUREMENTS IN A TURBULENT BOUNDARY LAYER"

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IN A RECENT paper, Shlien and Corrsin [1] presented some interesting measurements of statistically steady temperature fields in a turbulent boundary layer. In each case the temperature field was generated by steadily heating a thin platinum wire mounted transverse to the mainstream and parallel to the wall on which the boundary layer was formed (by sanding paper located a substantial distance upstream from the wire). Some of the measurements were interpreted in terms of a theory by Batchelor [2], and it is with this interpretation that this letter is entirely concerned.

Granted the specific aim of this letter it is simplest to use a somewhat different notation from Shlien and Corrsin. Take x and y to be distances measured downstream from the heated wire, and normal to the wall, respectively. Thus if y_T is the distance of the wire from the wall, its position is $x = 0$, $y = y_T$. Among the measurements reported by Shlien and Corrsin were plots of the ensemble mean temperature field $\bar{\theta}(x, y)$ for four values of y_T. Particularly relevant to this letter are graphs (derived from these plots) of $\bar{y}(x)$ and $\sigma^2(x)$ defined by the equations

$$\bar{y}(x) = \int_0^\infty y\bar{\theta}(x, y)\,dy \Big/ \int_0^\infty \bar{\theta}(x, y)\,dy; \tag{1}$$

$$\sigma^2(x) = \int_0^\infty (y-\bar{y})^2\bar{\theta}(x, y)\,dy \Big/ \int_0^\infty \bar{\theta}(x, y)\,dy. \tag{2}$$

Thus $\bar{y}(x)$ and $\sigma^2(x)$ are respectively the centroid and variance of the mean temperature field at a fixed distance x downstream from the wire.

The theory of Batchelor [2] is concerned with a statistically unsteady problem in contrast with the statistically steady problem investigated by Shlien and Corrsin. Using the notation introduced above, consider a heated fluid particle released from $x = 0$, $y = y_T$ at $t = 0$, and let its position at time $t > 0$ be $x = X(t)$, $y = Y(t)$. Batchelor was concerned with the way in which the statistical properties of $X(t)$ and $Y(t)$ change with time in a situation when the following conditions are satisfied:

(i) the ensemble mean Eulerian velocity in the boundary layer $\bar{u}(y)$ is given by

$$\bar{u}(y) = \frac{u_*}{\kappa}\log\frac{y}{y_0},$$

where u_* is the friction velocity, y_0 is the equivalent roughness height and κ is von Kármán's constant.

(ii) the Eulerian velocity fluctuations in the boundary layer have steady statistical properties dependent only on u_* and y.

(iii) molecular transfer processes and buoyancy forces are negligible.

(iv) t is small enough for the heated fluid particle to remain within the region where (i) and (ii) are satisfied for all but a negligible fraction of the realizations of its motion. [It is implied by (iii) and (iv) that any viscous or conducting sublayer has a negligible effect on the statistical properties of $X(t)$ and $Y(t)$.] Under these conditions Batchelor showed, without assuming the existence of an eddy viscosity or an eddy diffusivity, that there are universal constants b and c such that, when $y_T = 0$,

$$\overline{Y}(t) = bu_*t, \tag{3}$$

$$b\kappa\overline{X}(t) = \overline{Y}(t)\left[\log\left(\frac{c\overline{Y}(t)}{y_0}\right) - 1\right], \tag{4}$$

where the overbars denote ensemble means. When $y_T \neq 0$, equations (3) and (4) hold approximately provided $u_*t \gg y_T$. When the further hypotheses are made that an eddy viscosity and an eddy diffusivity exist and that they are equal, it was shown by Chatwin [3] that

$$b = \kappa \approx 0.41, \quad c \approx 0.56,$$

and that, consistent with the similarity arguments used by Batchelor,

$$\overline{(X-\overline{X})^2} \approx 3.84u_*^2t^2, \quad \overline{(Y-\overline{Y})^2} \approx 0.17u_*^2t^2. \tag{5}$$

(Note that there is a misprint in the footnote on p. 292 of [1].)

In attempting to interpret results of statistically steady experiments like those presented in [1] in terms of the theoretical predictions for statistically unsteady quantities just described, it is first necessary to determine that part of the heated fluid, if any, within which all but a negligible fraction of the fluid particles have moved, since release, so that Batchelor's theory can be applied. Shlien and Corrsin showed (in the first paragraph on p. 292 of [1]) that there was no such part of the heated fluid for three of the four values of y_T used in their experiments, either because the source was near the edge of the boundary layer [so that condition (ii) was not satisfied] or because observations could not be made at a sufficient distance downstream from the source for the particles to have forgotten where they started from [so that u_*t was not much greater than y_T]. For this reason the comparison with Batchelor's results which Shlien and Corrsin made when y_T was 1.66 displacement thicknesses was invalid, as indeed they themselves point out. Only for those of their experiments with $y_T = 0$ was a possible comparison indicated, and then only for that part of the heated fluid extending up to 40 displacement thicknesses downstream from the source. [It is therefore difficult to understand how Shlien and Corrsin could claim from their Fig. 16 that observations of $\sigma^2(x)$—see equation (2) above—taken up to 150 displacement thickness downstream contradicted equation (5), but that is not the main point of this letter.]

The assumption that Shlien and Corrsin made in order to compare their results for $y_T = 0$ with Batchelor's theory was that the graph of $\bar{y}(x)$ against x, where $\bar{y}(x)$ is defined in terms of $\bar{\theta}(x, y)$ by equation (1), was identical with the graph of $\overline{Y}(t)$ against $\overline{X}(t)$ obtained from equation (4). The same assumption has in effect been made by other workers including Gifford [4] and Cermak [5]. It is rather difficult to see any firm basis for this assumption since the two relationships describe different situations. For the value of $\bar{y}(x)$ at a given value of x is an average over fluid particles with different times of travel since release from the source but the same downstream displacement, while the value of $\overline{Y}(t)$ at a given value of $\overline{X}(t)$ [and therefore of t, but *not* of $X(t)$] is an average over fluid particles with different downstream displacements. For a given value of x, it is to be expected that most of those particles further from the wall than $\bar{y}(x)$ will have left the source later (since they are travelling quicker) than most of those particles nearer the wall than $\bar{y}(x)$, but it is not obvious why these two effects should cancel *exactly*, which is equivalent to what Shlien and Corrsin assumed. The two different graphs might have the same general shapes, but without additional argument it is not possible to justify the use of the graph of $\bar{y}(x)$ against x to determine the constants b and c appearing in equations (3) and (4).

These comments can be illustrated from the results of a theory by Townsend [6, 7] which is summarized on p. 361–364 of his book [8]. Townsend argued that, in the type of experiment considered by Shlien and Corrsin [supposing as always that conditions (i) to (iv) above hold], $\bar{\theta}(x, y)$ is approximately self-similar with the form

$$\bar{\theta}(x, y) = -\frac{Q}{\kappa_0 u_*}\left[\frac{1}{l}\frac{\mathrm{d}l}{\mathrm{d}x}\right]\eta f'(\eta) \text{ where } \eta = \frac{y}{l(x)}, \quad (6)$$

with Q being the strength of the line source, κ_0 being a constant (equal to b and κ if Reynolds analogy holds) and $f(\eta)$ is an unknown decreasing function whose integral from 0 to ∞ is 1. Townsend showed that equation (6) is consistent with the governing equations for large l/y_0 provided

$$l\left[\log\left(\frac{l}{y_0}\right) + \int_0^\infty f(\eta)\log\eta\,\mathrm{d}\eta\right] = \kappa\kappa_0 x. \quad (7)$$

Using equations (1) and (6) it follows that in Townsend's theory

$$\bar{y}(x) = l(x)\frac{\int_0^\infty \eta^2 f'(\eta)\,\mathrm{d}\eta}{\int_0^\infty \eta f'(\eta)\,\mathrm{d}\eta} = 2l(x)\int_0^\infty \eta f(\eta)\,\mathrm{d}\eta, \quad (8)$$

using the above condition on the integral of f from 0 to ∞. Townsend considered three reasonable particular forms for f which lead to the following results for $l(x)$ and $\bar{y}(x)$:

$$-\eta f' = \begin{cases}1, & \eta < 1\\ 0, & \eta > 1\end{cases}; \quad l\left[\log\left(\frac{0.37l}{y_0}\right) - 1\right] = \kappa\kappa_0 x; \quad \bar{y}(x) = \tfrac{1}{2}l(x); \quad (9a)$$

$$-\eta f' = \mathrm{e}^{-\eta}; \quad l\left[\log\left(\frac{0.56l}{y_0}\right) - 1\right] = \kappa\kappa_0 x; \quad \bar{y}(x) = l(x); \quad (9b)$$

$$-\eta f' = \begin{cases}1 - \tfrac{1}{2}\eta, & \eta < 2\\ 0, & \eta > 2\end{cases}; \quad l\left[\log\left(\frac{0.45l}{y_0}\right) - 1\right] = \kappa\kappa_0 x; \quad \bar{y}(x) = \tfrac{2}{3}l(x). \quad (9c)$$

Thus the relation between $l(x)$ and x in each of the equations (9a) to (9c) is of the same form as the relation between $\bar{Y}(t)$ and $\bar{X}(t)$ in equation (4) (and indeed all four relations are approximately the same far from the source). However, and this strikingly illustrates the principal point of the previous paragraph, the ratio of $\bar{y}(x)$ to $l(x)$ varies in the three examples from $\frac{1}{2}$ to 1. Evidence quoted in [8] suggests that the true distribution is intermediate between those in equations (9b) and (9c).

P. C. Chatwin

Department of Applied Mathematics and Theoretical Physics
The University of Liverpool
Liverpool L69 3BX
U.K.

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SPECIFICATION OF THE CORRECT BOUNDARY CONDITIONS

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NOMENCLATURE

- k, thermal conductivity [W/cm°C];
- L, length of the cylinder [cm];
- (Q/A), heat flux at the wall [W/cm^2];
- R, cylinder radius [cm];
- r, radial location [cm];
- T, temperature [°C];
- z, axial location [cm].

In a paper published recently, Archambault and Chevrier [1] used an implicit numerical technique based on the superposition principle to solve the two-dimensional unsteady state diffusion equation for a cylinder. This method is mathematically rigorous for a linear system and appears to have been used for the first time in connection with a numerical solution to a heat conduction problem. However, two errors were introduced in their paper; and, moreover, they did not indicate that they had checked their approximate solution against a known solution. Since we have been investigating a similar problem in connection with boiling around large horizontal cylinders [2], we report on this work as it relates to the Archambault and Chevrier paper. First, the authors failed to recognize that,

$$\text{as } r \to 0 \qquad \frac{1}{r}\frac{\partial T}{\partial r} = \frac{\partial^2 T}{\partial r^2} \quad (1)$$

which makes the first part of the diffusion equation equal to:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = 2\frac{\partial^2 T}{\partial r^2} \quad \text{at} \quad r = 0. \quad (2)$$